ملاحظة: يمنع استخدام الآلة الحاسبة

مجموع درجات هذا الامتحان 60 درجة

Question 1:

Choose the right answer: **[1 mark each]**

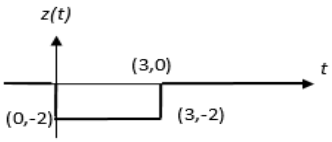
a. أي من الإشارات التالية هي إشارة طاقة:

.1 r(t)-r(t-2)	.2 u(t)-u(t-10)
.3 sin (3t)	.4 u(t)

b. $\int_{-1}^{5} e^{-4t^2}\, cos(t^2)\delta(t-10)dt =$

.1 $e^{-400}cos(100)$	.2 $e^{-400}cos(100)\delta(t-10)$
.3 $e^{-400}sin(100)$	.4 0

c. أي من الصيغ الرياضية الاتية تصف الإشارة z(t) التي بالشكل:



.1 -u(t)+u(t-3)	.2 -2u(t).[u(-t+3)]
.3 $-2\text{rect}(\frac{t}{3}-0.5)$	.4 -2 sinc (3t)

d. الإشارة x(t)– هي انعكاس الإشارة x(t)حول:

.1 المحور الرأسي	.2 المحور الأفقي
.3 نقطة الاصل	.4 كل الإجابات خطأ

e. الزمن الدوري الرئيسي للإشارة $x(t)= 2\cos(t + \pi/4) + 4\sin(\pi t/3)$ **هو :**

.1 2π	.2 3
.3 6π	.4 كل الإجابات خطأ، الإشارة غير دورية

f. أي من العبارات الاتية صحيح:

.1 كل الأنظمة السببية أنظمة بدون ذاكرة	.2 كل الأنظمة التي بدون ذاكرة أنظمة سببية
.3 كل الأنظمة التي بدون ذاكرة أنظمة غير سببية	.4 كل الأنظمة غير السببية أنظمة بدون ذاكرة

g. أي من الأنظمة الاتية هو نظام سببي:

.1 $y(t)= x(-t)$	.2 $y(t)= x(t)$+t-1
.3 $y(t)= x(t^{1/2})$	.4 $y(t)= x(t+5)$-5

h. النظام المحدد بالعلاقة $y(t) = x(4 - t)$ هو نظام:

1. متغير مع الزمن	2. سببي
3. بدون ذاكرة	4. كل الإجابات خطأ

i. أي من تحويلات فوريير الآتية صحيح:

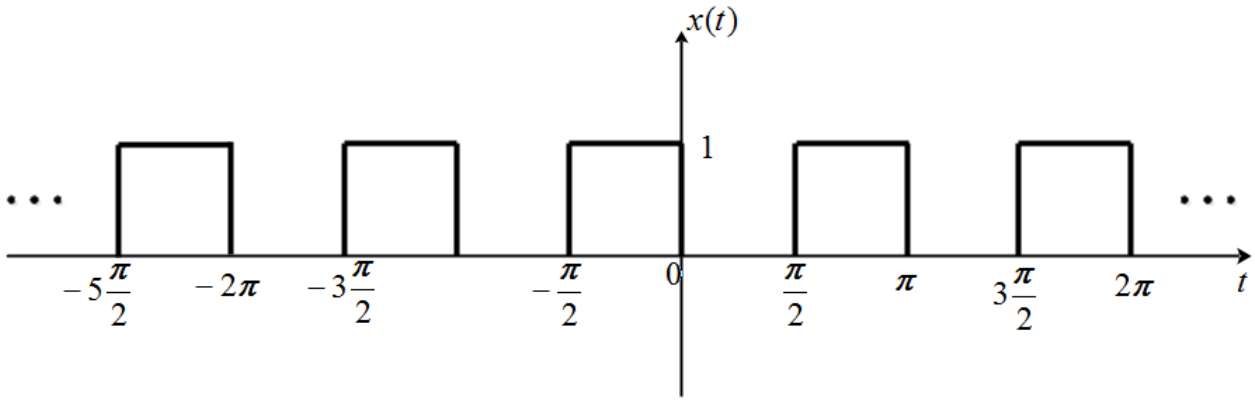
1. $\cos(\pi t) \overset{F.T}{\leftrightarrow} \frac{\pi}{j}[\delta(\omega-\pi)+\delta(\omega+\pi)]$	2. $2\delta(t) \overset{F.T}{\leftrightarrow} 2\pi$
3. $\sin(\pi t) \overset{F.T}{\leftrightarrow} \frac{\pi}{j}[\delta(\omega-\pi)+\delta(\omega+\pi)]$	4. $2\delta(t) \overset{F.T}{\leftrightarrow} 2$

j. تحويلة فوريير للإشارة $e^{-3t}u(t)$ هي:

1. $X(\omega) = \frac{1}{3-j\omega}$	2. $X(\omega) = \frac{1}{3+j\omega}$
3. $X(\omega) = \frac{3}{9+\omega^2}$	4. $X(\omega) = \frac{6}{9+\omega^2}$

Question 2:

i. Compute and sketch the spectrum of the first 5 components of the trigonometric Fourier series for the waveform shown below. **[8]**



ii. A Fourier series of a periodic signal is given by:

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{1-(-1)^{n+1}}{n\pi}\right]\sin(200\pi nt)$$

a. Find the fundamental frequency in Hertz. **[1]**

b. Find the average (DC) value of the signal. **[1]**

c. Sketch the magnitude spectrum of the first (7) coefficients of the signal showing all important values on the sketch. **[2]**

d. Find the power of the signal using the first (7) coefficients. **[2]**

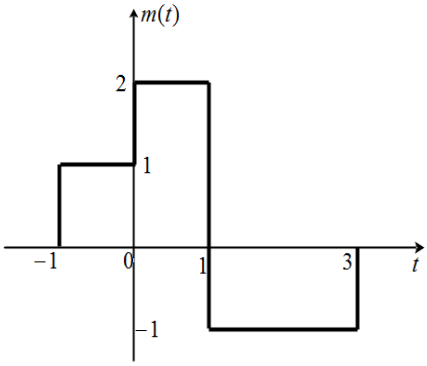
Question 3:

i. Using tables and specified property, Find the Fourier transform of :

[4 marks each]

a. $x(t) = \frac{8}{4+t^2}$ **(Use duality)**

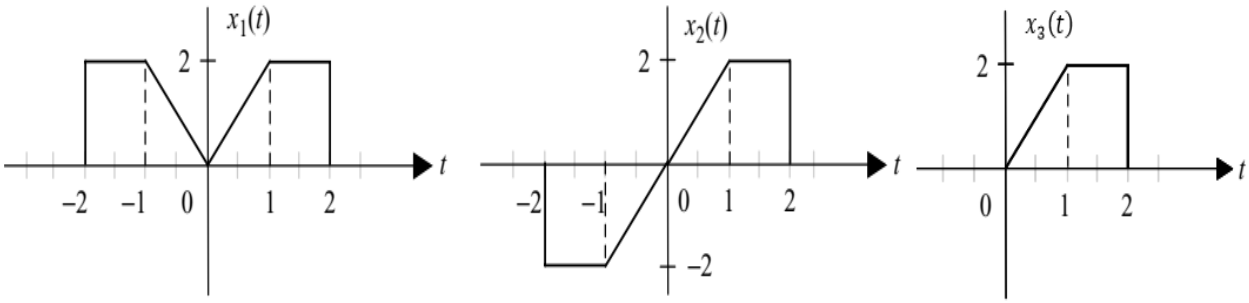
b. The shown signal $m(t)$ **(Use time differentiation)**



ii. Using tables and properties, find the Fourier transform and sketch the spectrum of:

$s(t) = cos(2t).cos(10t)$ **[4]**

iii. For the shown signals $x_1(t), x_2(t)$ *and* $x_3(t)$: **[4]**



If $\boldsymbol{X_1(\omega) = \frac{4}{\omega^2}[\omega \sin(2\omega) + \cos(\omega) - 1]}$

And $\boldsymbol{X_2(\omega) = j\frac{4}{\omega^2}[\omega \cos(2\omega) - \sin(\omega)]}$

Find $\boldsymbol{X_3(\omega)}$

Question 4:

i. Find the Laplace Transforms of the following function: **[4]**

$$\boldsymbol{f(t) = [t^3 + 3t^2 + 4t + 3\ ]u(t)}$$

i. Compute the inverse Laplace Transforms of the following function:

[4 marks each]

a. $X(s) = \dfrac{4}{(s+1)(s+3)}$ **b.** $F(s) = 9 + \dfrac{s^2+1}{s^2+4}$

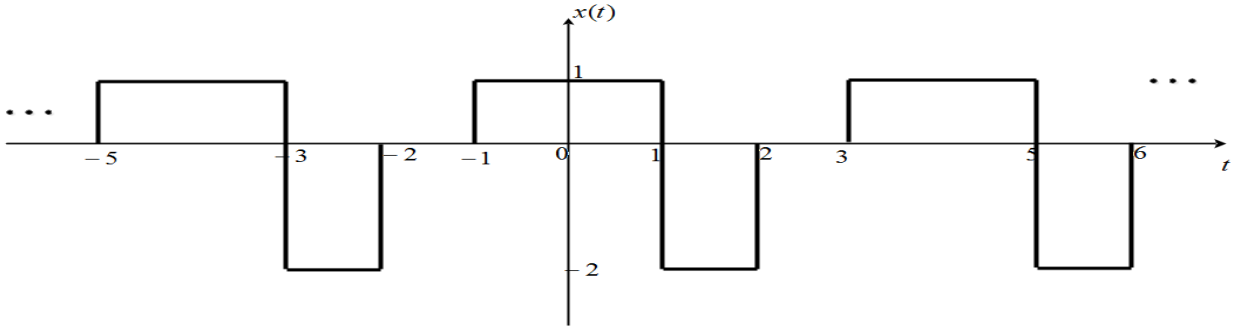
ii. Solve the following differential equation using the Laplace transform method

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = 2e^{-t}$$

If $y(0^-) = 2$, *and* $y'(0^-) = 2$ **[8]**

Question 5:

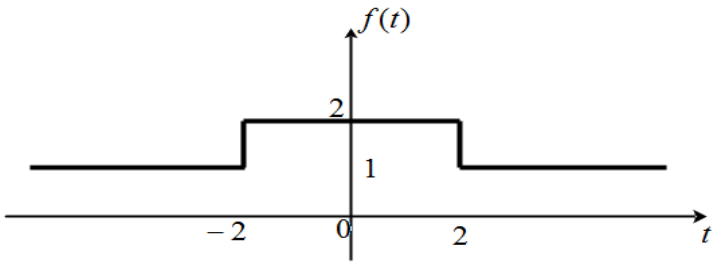
i. Compute and sketch the spectrum of the first 5 components of the trigonometric Fourier series for the waveform shown below. **[8]**



ii. Using tables and properties, find the Fourier transform and sketch the spectrum of: **[4]**

$$y(t) = [\frac{2}{4+jt}].cos(100t)$$

iii. Using tables and properties, find the Fourier transform of the following signal: **[4]**



iv. Using tables and **time differentiation property,** Find the Fourier transform of:

$$f(t) = -3[u(t+2) - u(t)] + 3[u(t) - u(t-2)]$$ **[4]**

Transformations' table:

$x(t)$	$X(\omega)$	$X(S)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	……
$sgn(t)$	$\frac{2}{j\omega}$	……
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	$\frac{1}{s}$
$e^{-at}u(t)$	$\frac{1}{j\omega+a}$	$\frac{1}{s+a}$
$cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$\frac{s}{s^2+\omega_0^2}$
$sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$\frac{\omega_0}{s^2+\omega_0^2}$
$P_t=\begin{cases}A & \lvert t\rvert < a\\ 0 & \lvert t\rvert > a\end{cases}$ Pulse Duration=2a Or $A\,rect(\frac{t}{2a})$	$2Aa\frac{\sin(\omega a)}{\omega a}$	……
$te^{-at}u(t)$	$\frac{1}{(j\omega+a)^2}$	$\frac{1}{(s+a)^2}$
$t^n e^{-at}u(t)$	$\frac{n!}{(j\omega+a)^{n+1}}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	……

Properties' table:

Property	Time Domain	Fourier Domain	Laplace Domain
Time shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-st_0}X(s)$
Frequency Shifting	$e^{j\omega_0 t}x(t)$ or $e^{s_0 t}x(t)$	$X(\omega-\omega_0)$	$X(s-s_0)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$	$sX(s)-x(0^-)$
	$\frac{d^2x(t)}{dt^2}$	$(j\omega)^2X(\omega)$	$s^2X(s)-sx(0^-)-x'(0^-)$
Frequency differentiation	$-tx(t)$	$-j\frac{dX}{d\omega}$	$\frac{dX}{ds}$
Duality	$X(t)$	$2\pi X(-\omega)$	……
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$	$X_1(s)X_2(s)$

Trigonometric Fourier Series	Exponential Fourier Series
$x(t)=a_0+\sum_{n=1}^{\infty}[a_n\cos(n\omega t)+b_n\sin(n\omega t)]$	$x(t)=\sum_{n=-\infty}^{\infty}D_n e^{jn\omega t}$
$a_0=\frac{1}{T_0}\int_{T_0}x(t).dt$	$D_n=\frac{1}{T_0}\int_{T_0}x(t)e^{-jn\omega t}.dt$
$a_n=\frac{2}{T_0}\int_{T_0}x(t)\cos(n\omega t).dt$	$D_n=\frac{a_n-jb_n}{2}$
$b_n=\frac{2}{T_0}\int_{T_0}x(t)\sin(n\omega t).dt$	$P_x=\sum_{n=-\infty}^{\infty}\lvert D_n\rvert^2=\lvert D_0\rvert^2+2\sum_{n=1}^{\infty}\lvert D_n\rvert^2$